Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42351201_OC**
Name of Paper	: **C 3-Calculus & Geometry**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Use (ε, δ) $definition\ to$ find δ such that $\lim_{x\to 3}(5x-2) = 13$; $\varepsilon = 0.01$.

Examine the continuity of the function $f(x) = \begin{cases} \cos x\,, & x \neq \frac{\pi}{2} \\ 1 \quad , & x = \frac{\pi}{2} \end{cases}$

at $x = \frac{\pi}{2}$.

Show that the function

$g(x) = \begin{cases} x\ sin\frac{1}{x}, & when\ x \neq 0 \\ 0 \quad , & when\ x = 0 \end{cases}$ is continuous everywhere.

2. Discuss the continuity and differentiablity of the function $f(x) = (2x-3)^{\frac{5}{2}}$ $at\ x = \frac{3}{2}$.

Show that the function $f(x) = e^{-|x|}$ and $g(x) = x + |x|$ are not differentiable at $x = 0$.

Verify the lagrange's mean value theorem for the function $f(x) = \sqrt{25 - x^2}$ in the interval [1,5].

3. Find the asymptotes of the curve $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$.

Find the integration of the function $f(x) = \sqrt{x^2 + 4x - 5}$.

Derive the formula for the volume of a sphere of radius r.

4. Describe the graph of the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

Find the equation for the ellipse with foci $(0, \pm 6)$ and length of minor axis 16.

5. Trace the conic $16x^2 - 24xy + 9y^2 + 110x - 20y + 100 = 0$ by rotating the coordinate axes to remove the xy -term.

Let $\boldsymbol{r}(t) = 2t\,\boldsymbol{i} + 3t^2\boldsymbol{j} + t^3\boldsymbol{k}$. Find $\lim_{t\to 2} \boldsymbol{r}(t).\left(\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)\right)$.

Find a vector of length $\sqrt{17}$ that makes an angle of $\pi/6$ with the positive x -axis.

6. Sketch the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$.

For $\boldsymbol{A} = 2x^2\boldsymbol{i} - 3yz\,\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\varphi = 2z - x^3y$, find $\boldsymbol{A}.\boldsymbol{\nabla}\varphi$ and $\boldsymbol{A} \times \boldsymbol{\nabla}\varphi$ at the point (2, -1, 1).

If $\boldsymbol{A} = 2yz\,\boldsymbol{i} - x^2y\,\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\varphi = 2x^2yz^3$, find $(\boldsymbol{A}.\boldsymbol{\nabla})\varphi$ and $\boldsymbol{A}.(\boldsymbol{\nabla}\varphi)$.